



Digital Logic Design

Lecture 4

Dr. Navabi

2 Lecture 4 (Karnaugh)
2-Var Maps
Definitions
3-Var Maps
4-Var - - -

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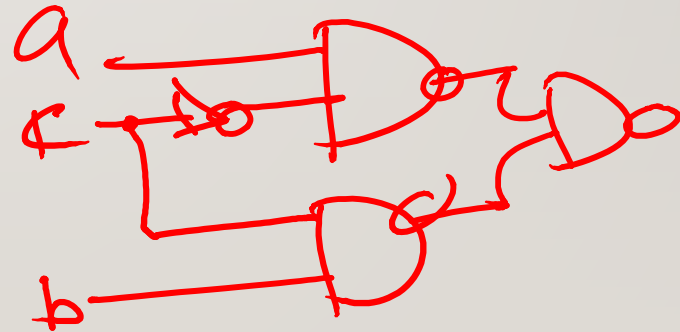
$$w = a\bar{c} + bc + \underline{ab}$$

$$= a\bar{c} + bc + ab(c + \bar{c})$$

$$= a\bar{c} + bc + abc + ab\bar{c}$$

$$= a\bar{c}(1+b) + bc(1+a)$$

$$= a\bar{c} + bc$$



$$4 \quad w = cd + a\bar{c}d + \bar{a}c\bar{d} + ab\bar{c} + \bar{a}b\bar{d}$$

$$= cd(a + \bar{a}) + a\bar{c}d + \bar{a}c\bar{d} + ab\bar{c}(d + \bar{d}) + \bar{a}b\bar{d}(c + \bar{c})$$

$$= \underbrace{acd + \bar{a}cd}_{ad(c + \bar{c})} + \underbrace{a\bar{c}d + \bar{a}c\bar{d}}_{\bar{a}c(d + \bar{d})} + \underbrace{ab\bar{c}d + ab\bar{c}\bar{d}}_{ab\bar{c}(d + \bar{d})} + \underbrace{\bar{a}b\bar{c}\bar{d} + \bar{a}b\bar{d}\bar{c}}_{\bar{a}b\bar{d}(c + \bar{c})} =$$

$$= ad(c + \bar{c}) + \bar{a}c(d + \bar{d})$$

$$= ad + \bar{a}c + ab\bar{c}d + \bar{a}b\bar{c}\bar{d} + b\bar{c}\bar{d}$$

$$= ad(1 + b\bar{c}) + \bar{a}c(1 + b\bar{d}) + b\bar{c}\bar{d} = ad + \bar{a}c + b\bar{c}\bar{d}$$

$$b\bar{c}\bar{d}(a + \bar{a})$$

$$b\bar{c}\bar{d}$$

5 Karnaugh Map

$$\underline{\bar{a}}\underline{\bar{b}}(\underline{c}d + \underline{a}\underline{\bar{b}}\underline{\bar{c}}d)$$

$$\bar{a}\bar{b}d(c + \bar{c}) \rightarrow \bar{a}\bar{b}d$$

	ab	w
0:	00	0
1:	01	1
2:	10	1
3:	11	1



$$w = \underline{\bar{a}}\underline{b} + \underline{a}\underline{\bar{b}} + \underline{a}\underline{b}$$

$$= \sum_{m=1,2,3}$$

Sum of Products = SOP

product
multiplication

$$\bar{a}b + a\bar{b} + ab$$

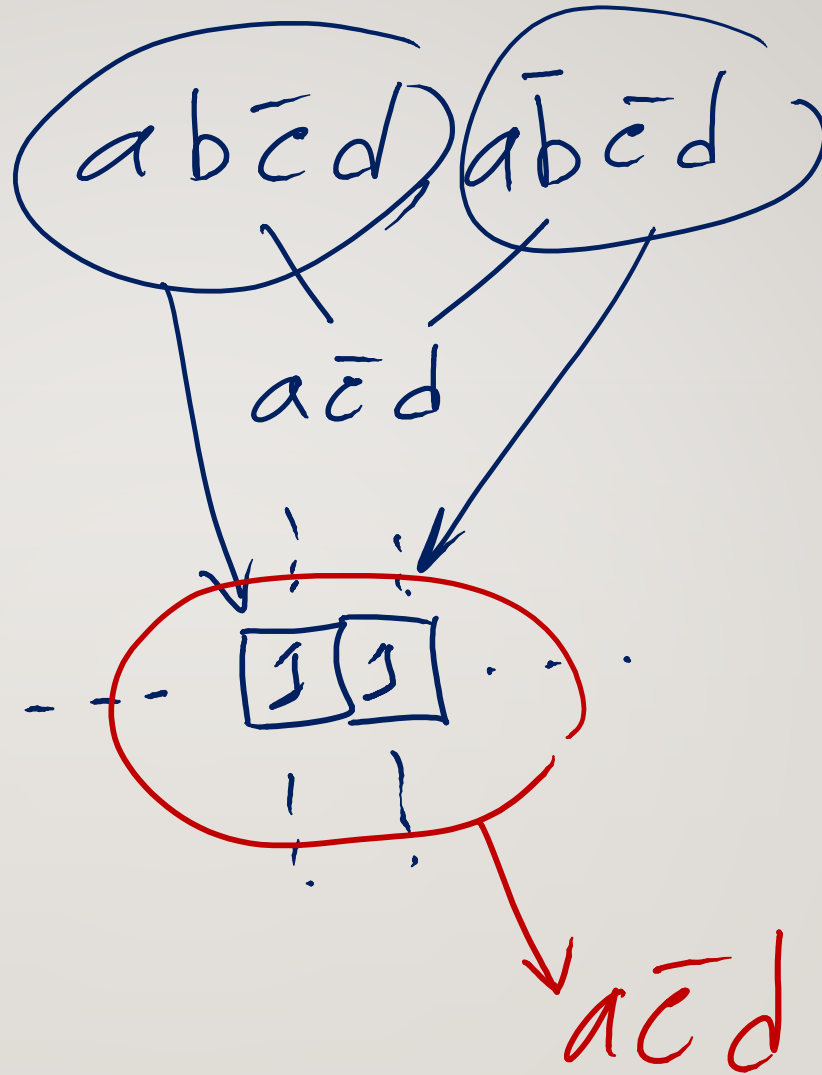
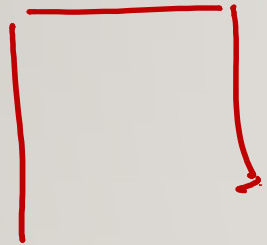
$$= \bar{a}b + ab + a\bar{b} + ab$$

$$= b(\bar{a} + a) + a(\bar{b} + b)$$

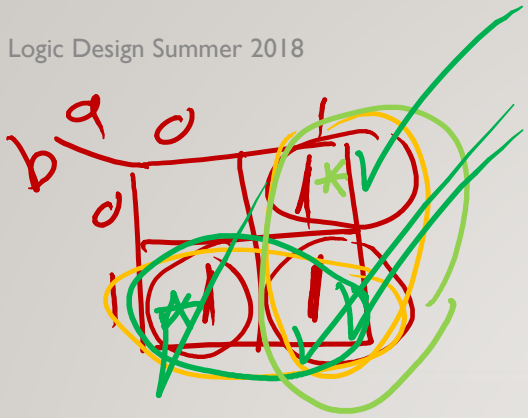
$$= b + a$$

6 Boolean adj

Physical "



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$a\bar{b}, \bar{a}b, ab, a, b$

a, b

a, b

Minterm: a prod term w/ all func variabls

Prod: any numb of var anded

Sum: " " " " +v term or'd

any product term than when 1, func is 1:

An imp not compl covered by any other Implicant
implicant is a PI

EPI: imp with at least a minterm not covered by any other PI

8 Min Coverage

1. List all EPJ ✓

2. any uncovered min?
exit if none ✓

3. Take PJ covering most #
of uncovered min terms

$$w = a + b$$

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	a	b	c	w
0:	0	0	0	0
1:	0	0	1	0
2:	0	1	0	0
3:	0	1	1	1
4:	1	0	0	1
5:	1	0	1	0
6:	1	1	0	1
7:	1	1	1	1

c \ ab	00	01	11	10
0				
1	x	⊗	⊗	

$\bar{a}\bar{b}c$ $\bar{a}bc$ $\bar{a}bc$ ~~$a\bar{b}c$~~

$\bar{a}c$

$\bar{a}bc$ abc
 $\bar{a}c + bc$

bc \ a	0	1
00	0: 4:	
01	1: 5:	
11	3: 7:	
10	2: 6:	

$\Sigma_m(3,4,6,7)$

c \ ab	00	01	11	10
0	0:	2:	⊗ 3: 4: 1*	
1	1:	⊗ 5: 7: 1*	6:	5:

10

$$F(a,b,c) = a\bar{c} + bc + ab$$

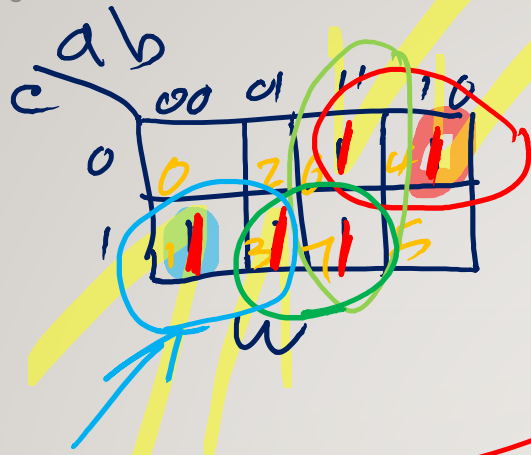


$bc, ab, a\bar{c}$

$$F = bc + a\bar{c}$$

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	a	b	c	w
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

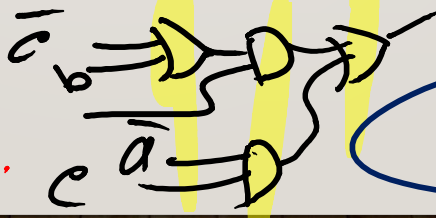


$$\bar{a}c + a\bar{c} + \begin{cases} ab \\ bc \end{cases}$$



$$\Sigma_m(1, 3, 4, 6, 7)$$

$$\bar{a}c + a\bar{c} + ab = \bar{a}c + a(\bar{c} + b)$$



$$\bar{a}\bar{b}, \bar{a}b, bc$$

$$\bar{a} + bc$$

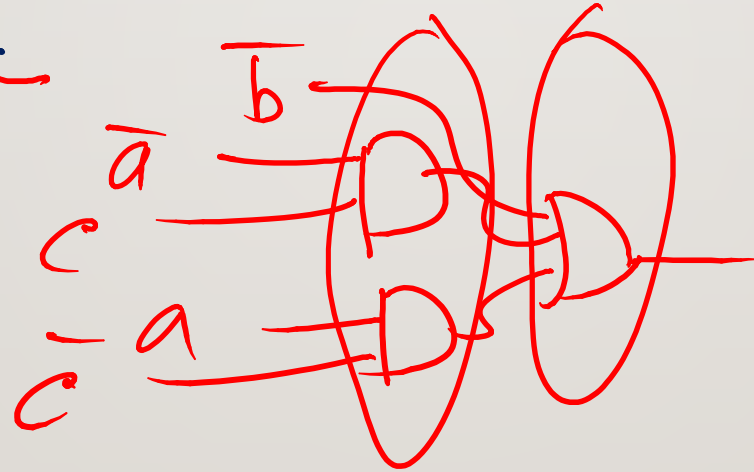
12

	a	b	c	d	11	10
0	1	1	1	1		
1		1				

$$w = \bar{c} + \bar{a}b$$

	a	b	c	d	11	10
0	*	1		1*	1	
1	1	*	1			*1

$$w = \bar{b} + \bar{a}c + ac\bar{c}$$



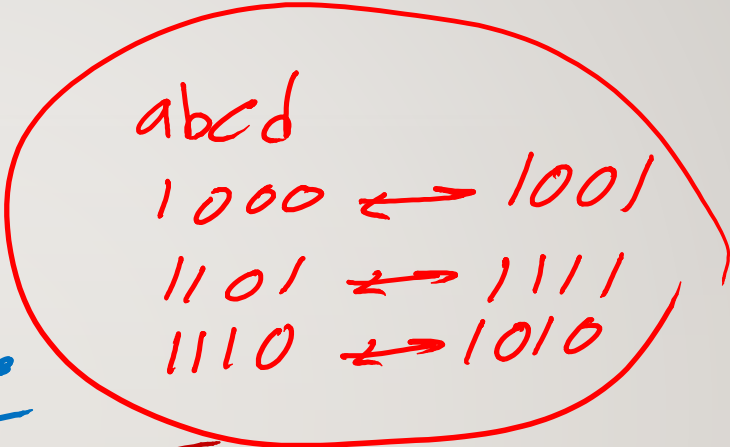
13

	a	b	c	d	w
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

cd	ab			
	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

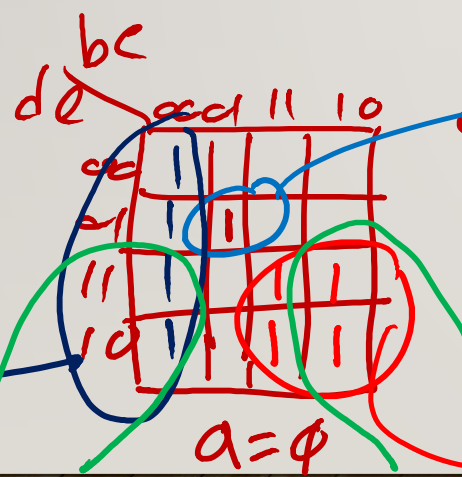


$$\bar{a}\bar{b} + \bar{a}\bar{c} + abd + \bar{b}\bar{c}$$



$$f = \bar{b}\bar{d} + a\bar{c}d + abc$$

5Var



$$\bar{b}c\bar{d}e$$

$$\bar{a}\bar{b}c\bar{d}e, a\bar{b}c\bar{d}e$$

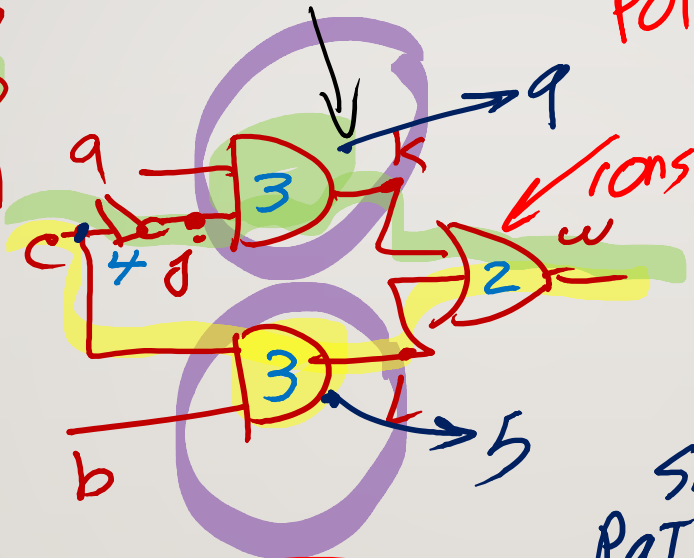
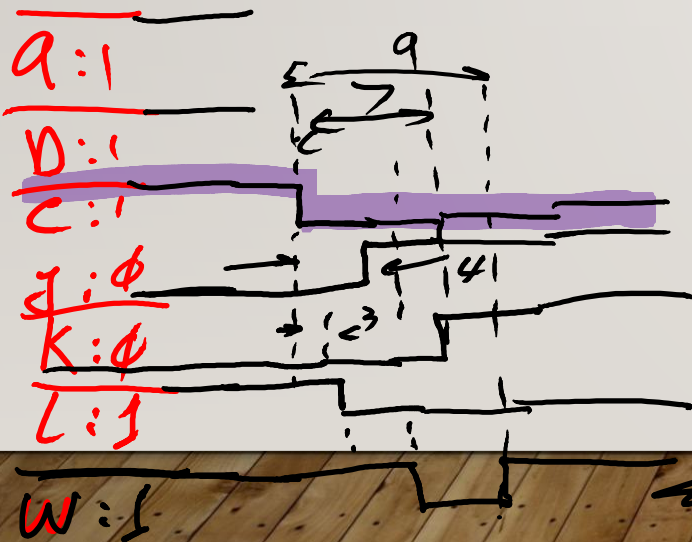
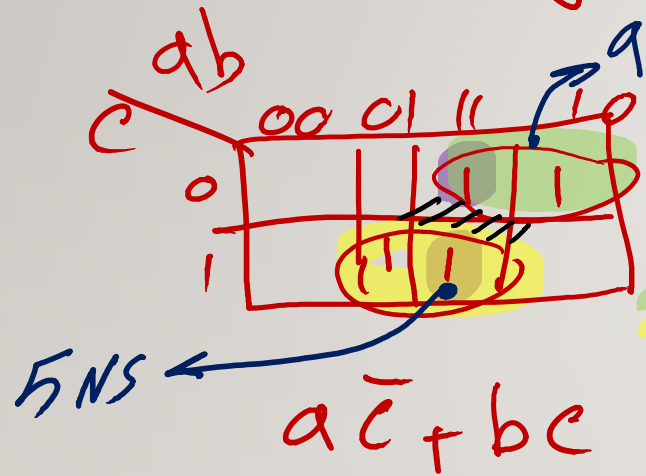
$$\bar{a}\bar{b}c$$

$$bd$$

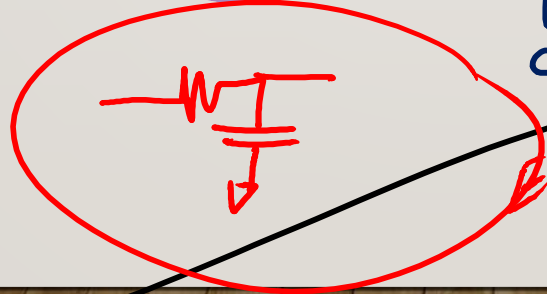
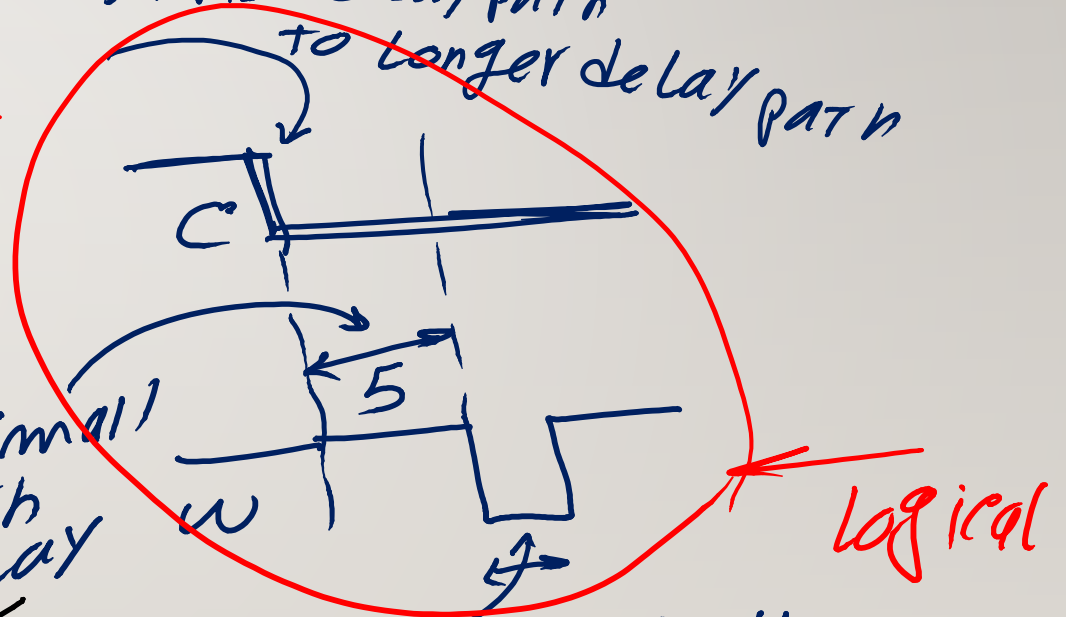
$$\bar{c}d$$

15 Timing: STATIC POTENTIAL Hazard

$\begin{matrix} a & b & c \\ 1 & 1 & 1 \end{matrix} \rightarrow 110$



shorter delay path to longer delay path



Longer del - Shorter delay

$a - 5 = 4$