Iran Center of ESL Education and Promotion

Haming Code

Contents

1- Introduction

In the late 1940s Richard Hamming recognized that the further evolution of computers required greater reliability, in particular the ability to detect and correct errors. (At the time, parity checking was being used to detect errors, but was unable to correct any errors.) He created the, Hamming Codes, perfect 1-error correcting codes, and the extended Hamming Codes, 1-error correcting and 2-error detecting codes.

2- Algorithm

The following general algorithm generates a single-error correcting (SEC) code for any number of bits.

1. Number the bits starting from 1: bit 1, 2, 3, 4, 5, etc.
2. Write the bit numbers in binary: 1, 10, 11, 100, 101, etc.
3. All bit positions that are powers of two (have only one 1 bit in the binary form of their position) are parity bits: 1, 2, 4, 8, etc. (1, 10, 100, 1000)
4. All other bit positions, with two or more 1 bits in the binary form of their position, are data bits.
5. Each data bit is included in a unique set of 2 or more parity bits, as determined by the binary form of its bit position.
    1. Parity bit 1 covers all bit positions which have the least significant bit set: bit 1 (the parity bit itself), 3, 5, 7, 9, etc.
    2. Parity bit 2 covers all bit positions which have the second least significant bit set: bit 2 (the parity bit itself), 3, 6, 7, 10, 11, etc.
    3. Parity bit 4 covers all bit positions which have the third least significant bit set: bits 4–7, 12–15, 20–23, etc.
    4. Parity bit 8 covers all bit positions which have the fourth least significant bit set: bits 8–15, 24–31, 40–47, etc.
    5. In general each parity bit covers all bits where the bitwise AND of the parity position and the bit position is non-zero.

The form of the parity is irrelevant. Even parity is simpler from the perspective of theoretical mathematics, but there is no difference in practice.

This general rule can be shown visually:

Bit position		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Encoded data bits		p1	p2	d1	p4	d2	d3	d4	p8	d5	d6	d7	d8	d9	d10	d11	p16	d12	d13	d14	d15
Parity bit coverage	p1	X		X		X		X		X		X		X		X		X		X	
	p2		X	X			X	X			X	X			X	X			X	X	
	p4				X	X	X	X					X	X	X	X					X
	p8								X	X	X	X	X	X	X	X					
	p16																X	X	X	X	X

Figure 1 - General Rule

Linear codes with length n and dimension k will be described as [n,k] codes. Hamming Codes are linear codes, and a Hamming Code will be described as a [n,k] q-ary Hamming Code, where q is the size of the base field, Fq . In other words an [n,k] q-ary Hamming Code is a linear subspace of the n-dimensional vector space over Fq .

The Hamming distance d H between any two words of the same length is defined as the number of coordinates in which they differ. It is easy to see the Hamming distance is a metric. Then any word within distance 1 to a codeword is, in fact, within distance 1 to a unique codeword. Thus if any Hamming codeword is transmitted and at most 1 error occurs then the original codeword is the unique codeword within distance 1 to the received word. Thus it is true that the minimum distance between any two Hamming codewords is ≥ 3, then it is true that Hamming Codes are 1-error correcting. Decoding any received word to this nearest Hamming codeword corrects any single error.

Suppose that x and y are binary Hamming codewords of distance 3. Then one of x or y has even parity and the other odd, say x has even parity. If x' and y' are the extended Hamming codewords obtained from adding a check digit. Then x 0 = 0 since x has even parity and y 0 = 1 since y has odd parity. The distance between x' and y' is one more than the distance between x y, so the minimum distance between codewords of an extended Hamming Code is 4. Now any received word with one error is distance 1 from a unique codeword, and a received word with 2 errors is not within distance 1 from any codeword. If a word is within distance 1 to a codeword then we decode the word to that codeword as before. If a word is not within distance 1 to any codeword, then we recognize that 2 errors have occurred and report that two errors have occurred.

3- References

[1] Moon, Todd K. (2005). Error Correction Coding. New Jersey: John Wiley & Sons. ISBN 978-0-471-64800-0.